CONCEPT OF PHASE-NOISE TUNING OF BIPOLAR VOLTAGE-CONTROLLED OSCILLATORS

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ABSTRACT

So far, oscillators have been designed to perform a specific task while the key parameters such as phase-noise and power consumption are set by the hardware design and not by the communication system in an adaptive way. As the communication channel is often subject to changes, the conditions under which the oscillator operates should be changed as well. Therefore, the concept of design for adaptivity must be considered as a cornerstone in the design of today’s telecommunication systems. Accordingly, the concept of phase-noise tuning, introduced in this paper, shows how oscillators can trade performance for power consumption in an adaptive way. The presented model of a quasi-tapped bipolar VCO reveals the resulting trade-offs between phase-noise and power consumption imposed by the design for adaptivity.

1. INTRODUCTION

Currently, analog RF front-end circuit design is aimed at fulfilling a set of specifications resulting from non-volatile conditions, where the telecommunication systems are expected to be functional. However, the fact that radio channels are not kept in vitro, and so the conditions under which the system is supposed to operate are not fixed but rather variant [1], must be taken into account as a new trajectory in a design procedure. Referred to as “structured electronic design for adaptivity”, this concept leads to a procedure not opting for a particular operating condition, but a set of conditions.

Likewise, for oscillators and oscillating systems, as of the key building blocks in the RF front-end, there is a possibility to trade phase-noise for power. This will lead to power savings being of importance not only for the oscillator but for the front-end system as a whole as well. Therefore, the introduced concept of phase-noise tuning, should open the “door” for a new design procedure if the adaptivity is of concern.

The analytical expressions, presented in this paper, show how the phase-noise and power consumption trade between each other in case of a quasi-tapped bipolar voltage-controlled oscillator (VCO).

This paper is organized as follows. A phase-noise model of a quasi-tapped bipolar VCO [2] is presented in Section 2. The concept of phase-noise tuning is the subject of Section 3, while the conclusions are summarized in Section 4.

2. PHASE-NOISE MODEL OF A QUASI-TAPPED VCO

Being considered as “the best” maximum performance solution [2], the quasi-tapped bipolar VCO beneficially uses the advantage of an increased voltage swing over the LC-tank, while at the same time the transistors of the oscillator’s active part remain far from heavy saturation. Also, when compared to non-tapped VCO’s, the base current shot-noise and noise of the biasing network are reduced with the square of the quasi-tapping factor.

Note that, quasi-tapping capacitances $C_A$ and $C_B$, introduced in this paper, serve to define the quasi-tapping ratio and not, as presented in some references [3], to facilitate biasing of the transistors in the active part of the oscillator. Unlike such a constellation, where the influence of these capacitors on both the LC-tank and the performances of the oscillator is fully neglected, in the upcoming analysis it will be shown that the role of the quasi-tapping capacitances $C_A$ and $C_B$, in the oscillator under consideration, is substantially different, and that they determine the performances of the oscillator, being phase-noise and power-consumption, equally with the other elements in the circuitry. Therefore, there is a claim on the quasi-capacitive tapping of the LC-oscillators as to be introduced by the authors.

A schematic of a quasi-tapped oscillator is shown in Fig. 1, with a cross-coupled transconductance amplifier as the active part. The simplified model of the oscillator is shown in Fig. 2, corresponding to a three-port oscillating system.

The relation between the parameters of the oscillator can be summarized as:

$$G_{TK} = \frac{1}{R_P} + \frac{R_L}{(\omega_L)^2} + R_C(\omega_C)^2$$

$$n = 1 + \frac{C_L}{C_B}, \quad G_M = g_m/2, \quad G_{M,TK} = G_M/n$$

(1)

(2)
\[ L_{TOT} = L, \quad C_{TOT} = C + \frac{C_A C_B}{C_A + C_B}, \quad \omega_0 = \frac{1}{\sqrt{L_{TOT} C_{TOT}}} \]  

(3)

where \( L \) is the inductance, \( C \) the capacitance, \( R_s, R_e \) and \( R_T \) their parasitic resistances, \( G_M \) the effective tank conductance, \( C_A \) and \( C_B \) are the quasi-tapping capacitances, \( n \) the quasi-tapping factor, \( G_{MK} \) the transconductance of the active part of the oscillator, \( g_m \) the transconductance of the bipolar transistors.

![Fig. 1: Quasi-tapped LC-oscillator (concept).](image1)

Fig. 1 Quasi-tapped LC-oscillator (concept).

![Fig. 2: Simplified model of a quasi-tapped oscillator.](image2)

Fig. 2 Simplified model of a quasi-tapped oscillator.

### 2.1. Noise analysis

In the following analysis it is assumed that the oscillator operates in a near linear fashion, such that the original noise close to the carrier contributes to a great extent to the total oscillator noise, compared to the other contributors, such as base-band noise and the one obtained after mixing from the other harmonics.

First, let us denote the main noise sources of a bipolar transistor as shown in Fig. 3a.

![Fig. 3: (a) Noisy oscillator model. (b) Noisy tank model.](image3)

Fig. 3 (a) Noisy oscillator model. (b) Noisy tank model.

In order to switch to the equivalent model of Fig. 3b, it is necessary to transform the indicated noise sources to the corresponding LC-tank, where \( Z(\Delta \omega) \) is the equivalent impedance of the ideal tank at an angular frequency \( \omega_0 + \Delta \omega \) and \( \omega_0 = 2\pi f_0 \).

\[ Z(\omega_0 + \Delta \omega) = -\frac{j\omega_0 L}{2\Delta \omega / \omega_0} \]  

(4)

For the sake of brevity, and because of the apparent symmetry, only one half of the oscillator is depicted. However, in the calculations to come, the oscillator as a whole is being analyzed.

The equivalent noise current spectral densities at the output of the oscillator from the above noise sources, at the frequency \( f_0 + \Delta f \), are found as follows:

\[ I(\omega) = 2I_{(T)}(\omega) + I_{(C)}(\omega) + I_{(G)}(\omega) \]  

(5)

\[ I_{(T)}(\omega) = 4KT\beta \]  

(6)

\[ I_{(C)}(\omega) = 2qI_{B} \]  

(7)

\[ I_{(G)}(\omega) = \frac{G_{MK}}{4K} \]  

(8)

where \( I_B \) is the base current shot-noise, \( I_C \) the collector current shot-noise, \( I_{(T)} \) the base thermal noise, \( k \) is Boltzmann’s constant and \( T \) the absolute temperature. Considered to be uncorrelated, all noise sources add to the equivalent one as given in Eq. (9) and Fig. 3b.

\[ I_{TOT}^2 = I_{TOT}^2 + I_{TOT}^2 + I_{TOT}^2 \]  

(9)

Accordingly, the equivalent output voltage noise spectral density, to be used in the derivation of the phase-noise, is given as:

\[ \tilde{V}_{TOT}^2 = \tilde{V}_{TOT}^2 \left| Z(\Delta \omega) \right|^2 \]  

(10)

\[ \tilde{V}_{TOT}^2 = K T \left( \frac{G_{MK}}{(\omega_0 C_{TOT})} \right)^2 \frac{(\omega_0^2 + \frac{qI_{C}}{4KTG_{MK}} + 2n^2r_{gb}G_{MK} + \frac{qI_B}{n^24KTG_{MK}} \right)^2} {1 + \frac{qI_{C}}{4KTG_{MK}} + 2n^2r_{gb}G_{MK} + \frac{qI_B}{n^24KTG_{MK}} \right)^2} \]  

(11)

\[ A_T = 2n^2r_{gb}G_{MK} + \frac{1}{n^2} \]  

(12)

From the start-up condition Eq. (2) and assuming \( \beta >> 1 \), factor \( A_T \) [4] – the noise factor of the active part – can be rewritten as:

\[ A_{T,s-up} = n(1/2 + r_{gb}g_{ms,s-up}) \]  

(13)

or for the safety start-up, corresponding to the case with the excess loop-gain larger than one – in this case \( k \) it is given as:

\[ A_{T,s-s-up} = n(k/2 + r_{gb}g_{ms,s-up}) \]  

(14)

Note, that indexes s-up and s-s-up correspond to the start-up and the safety start-up conditions of oscillators.

Now, let us denote \( L \) and \( V_S \) as phase noise and signal amplitude of the oscillator, where index \( QT \) is referred to a quasi-tapped oscillator. Defined as the ratio of the power in a 1Hz bandwidth at a frequency \( f_0 + \Delta f \) and the carrier power, the phase noise is given as:

\[ \text{Phase Noise} = \frac{1}{2} \left| A_{T,s-up} \right|^2 \]  

(15)
\[ L_{QT} = \frac{V_{N,TOT}^2}{V_{S,QT}^2} \]  
\[ L_{QT} \approx \frac{1 + 2n^2 r_b G_{BK} + G_{m,QT}/4G_{BK}}{V_{S,QT}^2} \]  

Combining Eqs. (11) and (15), the phase-noise of a quasi-tapped oscillator appears to be:

\[ L_{QT} \approx \frac{1 + 2n^2 r_b G_{BK} + G_{m,QT}/4G_{BK}}{V_{S,QT}^2} \]  

With the aid of Eq. (2) and for an arbitrary distance from the start-up condition, it can be written as:

\[ L_{QT} \approx \frac{1 + n(k/2 + r_b G_{m,QT})}{kT^2} \]  

where parameter \( k = G_{M,TK}/G_{BK} \) defines how far the oscillator is from the start-up condition. As this is a key parameter used in the following analysis, it is worth explaining, in more detail, its meaning and importance.

In its simplest form \( k \) equals the loop-gain of the oscillator as seen as a positive feedback amplifier. Also, it is the excess of the negative conductance, necessary for the compensation of the losses in the LC-tank. Namely, if the tank conductance is \( G_{BK} \), then for the start-up of the oscillations the equivalent negative conductance seen by the tank must be \( G_{M,TK} = kG_{BK} \), where \( k \) is larger than one, usually for a safety start-up set to a value of two. Note that expression (17) is not restricted to only one but to all the possible conditions under which the oscillator might operate.

### 3. PHASE-NOISE TUNING

Once the model of the oscillator is fully parameterized, it is possible to examine to what extent the phase-noise depends on its parameters. Accordingly, with the aid of Eq. (17), it is rather straightforward to examine how the phase-noise is changed if the power consumption is used as a parameter. Let us therefore call this phenomenon “phase-noise tuning”. The remainder of the section is fully in favor of this concept.

Nowadays, oscillators are designed so as to enable the complete front-end system to be functional under the most stringent conditions. However, if the conditions improve, it is, for example, no longer necessary to have the best possible phase-noise, but rather a lower one. This means that certain power savings would be possible as well. To respond to such a new situation, a design for adaptivity [5] appears to be a solution, as savings would be possible as well. To respond to such a new concept is addressed, it is necessary to broaden the meaning of parameter \( k \). As, in the oscillator under consideration, the start-up condition is also referred to as the minimum power condition, apart from defining how far the oscillator is from this state, parameter \( k \) also characterizes the increase in power. Namely, \( k \)-times larger negative conductance of active part of oscillator requires \( k \)-times increase in power with respect to the start-up condition. Note also, that the power is controlled by the tail current \( I_{TAIL} \), shown in the Fig. 1.

The next step to be performed in the analysis is to parameterize the amplitude of the signal, as it directly determines the range of operation of the oscillator. Defined as the product of the equivalent tank resistance at the resonant frequency and the first Fourier coefficient of the current sensed by the tank, the voltage swing over the tank equals:

\[ V_{S,UP,QT} = \frac{2I_{TAIL,UP}}{\pi G_{BK}}\]  

Combining this expression with Eq. (2), the start-up voltage swing over the tank is simply given as:

\[ V_{S,UP,QT} = \frac{8}{\pi} V_T \]  

where \( V_T \) is the thermal voltage, equal to \( KT/q \).

As for an arbitrary increase in power \( k \), \( V_{S,QT} \) equals

\[ V_{S,QT} = \frac{8}{\pi} n k V_T \]  

the most general expression for the phase-noise is:

\[ L_{QT} \approx \frac{1 + n(k/2 + r_b G_{m,QT})}{k^2 n^2} \]  

Now the phase-noise tuning range \( TR(k_1, k_2) \) for a \( k_2/k_1 \)-times increase in power, can be defined as:

\[ TR(k_1, k_2) = \frac{L_{QT}(k_1)}{L_{QT}(k_2)} = \frac{k_1^2}{k_2^2} \frac{1 + n(k_1/2 + c)}{1 + n(k_2/2 + c)} \]  

where \( c \) is a positive constant, defined as \( c = r_b G_{m,QT} \).

If it is known that minimum value for \( k \) is \( k_{MIN}=1 \) or for the safety start-up \( k_{MIN}=2 \), the question still remains what is the maximum value that parameter \( k \) might take. To answer this question, we should consider the mechanism that limits the performance of the oscillator. As it is the voltage swing across the resonator that can not increase beyond the limits imposed by the supply voltage and base-collector (BC) diode of the transistor, and in such a way puts a limit to the phase-noise, we will relate maximum value of \( k \) to this maximum voltage swing. Not only can the voltage swing increase over the limiting BC diode, but also for even lower voltages hard-saturation of the transistors keeps us far from operating in the vicinity of this point [6]. To reduce the detrimental effects of both hard-saturation and additional current noise of the forward biased BC diode, we will choose for the maximum voltage swing across the tank not to be larger than \( V_{MAX}=0.4 n V \). If the base-emitter voltage of transistor in active regime is round 0.7V and collector-emitter voltage in saturation round 0.3V, it is rather apparent how the value for the maximum voltage over the tank is obtained.

The fact that the bases of the transistors are connected to the supply voltage in the oscillator under consideration goes in line with the exposed reasoning, as well. Now, with the aid of Eq. (20), \( k_{MAX} \) has a value:

\[ k_{MAX} = \frac{0.4n}{8V_T} = 6 \]  

For example, if \( k_{MIN}=2 \) – the safety start-up – \( k_{MAX}=6 \) – expected maximum phase-noise – \( r_b \approx 40 \Omega \) and \( G_{m,QT} \) equals 7.6ms, the control range of the power consumption and the phase-noise, both for a tapping factor \( n=2 \), is:

\[ P_{MAX} = P_{MIN} \]  

\[ k_{MIN} = 3 \]

\[ 10\log[TR(2,6)] = 10\log\left[\frac{L_{QT,MIN}}{L_{QT,MAX}}\right] = 6.3dB \]  

where \( P_{MAX} \) and \( P_{MIN} \) represent maximum and minimum power consumption, while \( L_{MAX} \) and \( L_{MIN} \) represent the maximum and minimum phase-noise, corresponding to the values of \( k_{MAX} \) and \( k_{MIN} \), respectively.
Note, that in the extreme case, being phase-noise tuning from the start-up condition $k_{\text{MIN}}=1$ to $k_{\text{MAX}}=6$, and with the parameters the same as in the previous example, a phase-noise tuning range of 11 dB is achieved for a 6-times increase in power.

To illustrate how all the oscillator parameters of importance, being loop-gain, power consumption, phase-noise and signal amplitude, relate to each other, a unique presentation in the form of a $k$-rail diagram is introduced in Fig. 4.

The arrows in the diagram represent the lines of constant loop-gain, phase-noise and power consumption, respectively. In addition, from the $k$-rail diagram, it is seen that an increase in power results in a certain improvement of phase-noise, but only to a level determined by $k_{\text{MAX}}$. Increasing the loop-gain beyond this value leads only to a waste of power, as the phase-noise doesn’t improve any more. The corresponding phase-noise tuning range is shown in Fig. 4 between the points $PN_{\text{MIN}}$ and $PN_{\text{MAX}}$, as well.

It should be noted that the concept of “design for adaptivity” substantially differs from a standard, “fixed” design concept. Namely, as for the standard design it is satisfactory to choose for such a tank that the best phase-noise is achieved for $k_{\text{MAX}}=2$, in the design for adaptivity “the best” tank is chosen so as to obtain for $k_{\text{MAX}}$ as larger value as possible, for a given power budget.

Finally, note that both the exposed concept and the obtained results are fully confirmed by the CADENCE simulation tool SpectreRF.

4. CONCLUSIONS
Frequent changes of the conditions under which the communication systems of today operate, necessitate for a new approach in the design of analog RF front-end circuits. Structured electronic design for adaptivity is proposed as a new concept, introducing the design aspects that take into account a range of operating points rather than a fixed one.

Accordingly, as direct result of such a trend, the concept of “phase-noise tuning” is introduced in this paper. It explains how oscillators can trade performance for power consumption in an adaptive way. Applied to a quasi-tapped VCO, the analytical expressions present how the phase-noise changes with respect to the power consumption.

It has been shown what are the extremes, of the phase-noise tuning range and the range of power consumption, within which the oscillator is still functional. It has also been shown that unlike the standard design procedure, commonly aiming at the fixed loop-gain of two, a design for adaptivity aims at the best tank for the largest possible range of loop-gain values.

Finally, it is likely that the concept of design for adaptivity, and the corresponding concept of phase-noise tuning, lead to important power savings not only for the oscillator but for the front-end system as a whole as well.

5. REFERENCES