Spectral Analysis of Phase Noise in Bipolar LC-Oscillators—Theory, Verification, and Design
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Abstract—Interpretation of phase-noise generating mechanism in oscillators relies on approximate and numerical calculations for analysis, and simulation tools for synthesis. In this paper, a comprehensive, yet intuitive, phase-noise model is derived as a function of oscillator circuit parameters, suitting both analysis and synthesis of oscillators. Contributions of all noise sources to the phase noise of bipolar inductance–capacitance (LC) voltage-controlled oscillators have been determined using spectral noise analysis. For the verification of the phase-noise model, a bipolar LC-oscillator has been designed that realizes a phase noise of $-106$ dBc/Hz at 1 MHz offset from a 5.7 GHz-band carrier while drawing 4.8 mA from a 2.2 V supply. The results obtained from the phase-noise model agree well with simulation and measurement results of the LC-oscillator designed.

Index Terms—Duty cycle, noise factor, noise folding, phase noise, small-signal loop gain, voltage-controlled oscillator (VCO).

I. INTRODUCTION

Interpretations of the mechanism of phase-noise generation in oscillators are bounded to approximations and numerical calculations [1]–[8]. The phase-noise models obtained are often only indicative, failing to describe the oscillator phase noise using circuit parameters. Computer-aided simulation tools are therefore indispensable not only for synthesis but also for analysis of the oscillator performance.

In this paper, we introduce an intuitive, yet simple, phase-noise model for inductance–capacitance (LC) voltage-controlled oscillators (VCOs) using spectral noise analysis. It is amenable for design as it describes the noise performance of LC voltage-controlled oscillators qualitatively and quantitatively using electrical parameters.

Noise contributions to the phase noise of the LC-tank, transconductor ($g_m$-cell), and bias circuitry are determined for bipolar LC oscillators in this paper. The bias current-source noise of bipolar LC-VCOs is shown analytically to be larger than all other noise contributions put together (i.e., LC-tank noise and transconductor noise), in high-performance oscillator circuits [9], [10].

For the verification of the phase-noise model derived, a bipolar LC VCO has been designed. The oscillator simulation and measurement results have been shown to comply with the theoretical calculation results obtained from the phase-noise analysis method proposed.

A generic oscillator phase-noise model is described in the following section of this paper. Oscillator time-varying transfer functions are determined in Section III. Using spectral analysis of noise in oscillators, the contributions of all the noise sources to the phase noise are determined for bipolar LC-VCOs in Section IV, and the noise factors of the LC-tank, transconductor, and bias current source formulated. The phase-noise model derived is verified in Section V by comparing the theoretical and simulation results for the bipolar LC-oscillator. The test LC-oscillator circuit parameters and experimental results are presented in Section VI, confirming the validity of the phase-noise model. Paper is concluded in Section VII.

II. GENERIC PHASE-NOISE MODEL

In this paper, we will refer to the Bennett noise representation model [11], stating that if a signal is characterized by a single-sided power spectral density of $N$, then the amplitude of its component at a given frequency has a value of $\sqrt{2N}$. The relationships between single-sided and double-sided power spectral densities and corresponding Fourier magnitude components according to this model can be described as follows: for a $4KTR = N$ single-sided noise power spectral density of a resistor $R$, the noise signal magnitude at an arbitrary frequency $f_0$ of a polar Fourier development (positive frequencies) is $\sqrt{2KTR}$ (i.e., $\sqrt{2N}$), and of a complex Fourier development (positive and negative frequencies) is $\sqrt{2KTR}$ (i.e., $\sqrt{N/2}$) [12], $T$ being the absolute temperature, and $K$ Boltzmann’s constant.

Fig. 1(a) shows a block diagram of an LC-oscillator with a resonating LC-tank and a (nonlinear, limiting) cross-coupled transconductance amplifier (i.e., $g_m$-cell) in a positive feedback configuration over a voltage divider ($/n$).

$V_n$, $V_{(n)}$ and $V_{(n)}$ are the Fourier amplitude components of noise signals at the inputs of the transconductance amplifier, resonator, and feedback divider, respectively. For the amplitude of the signal current $i_S = V_{(n)}R_{TK}$ through the resonator, the phasor diagram shown in Fig. 1(b) illustrates the relationship between the noise component at a frequency $f_0 + \Delta$ and the oscillator signal at a frequency $f_0$. $V_S$ is the amplitude of the oscillation voltage signal across the LC-tank, $R_{TK}$ the LC-tank parallel loss resistance at resonance, $f_0$ the oscillation (resonant) frequency, and $\Delta$ the offset frequency. For the sake of a simpler notation, we treat as positive, by nature alternating phase-modulating components and noise conversion coefficients in the following analysis.

Referred to the oscillation signal component $i_S$, the noise component $i_{N,0}$ can be split into an in-phase $i_{\text{AM}}$...
Finally, the phase-related noise power density (thus referred to a 1 Hz bandwidth) relates to the single-sided phase noise ($\mathcal{L}$) at the output of the resonator at a frequency $f_0 + \Delta$ as

$$\mathcal{L} = \frac{|Z(f_0 + \Delta)|^2 \nu_{PM,TOT}^2}{v_S^2/2}.$$  (4)

$Z(f_0 + \Delta)$ is the equivalent $LC$-tank impedance at an offset frequency $\Delta$ from the resonant frequency $f_0$.

This is a well known phase-noise formulation, being the ratio of the noise power in a 1 Hz bandwidth at frequency $f_0 + \Delta$ and the carrier power at frequency $f_0$ [2]. For the oscillator model shown in Fig. 1(a) it reads

$$\mathcal{L} = \frac{\nu_{PM,TOT}^2}{v_S^2/2} = \frac{\nu_{PS}^2}{v_S^2/2} = |Z(f_0 + \Delta)|^2 \nu_{PM,TOT}^2$$  (5)

where $\nu_{PM,TOT}^2$ stands for the total voltage noise spectral density at the output of the oscillator (i.e., across the $LC$-tank).

In the remainder of this paper, the oscillator phase noise will be determined using the model shown in Fig. 1(a) and (1)–(5).

The transformations of the oscillator noise sources to the $LC$-tank ($i_{PM,TOT}$) must be known for an estimation of the oscillator phase noise. Considering the transconductor as a nonlinear voltage-to-current converter (i.e., limiter) [14] allows for the inclusion of all the noise generating mechanisms in $LC$-oscillators. By this, phenomena such as modulation of the transconductor noise and switching the noise of the bias current source, both resulting in the folding of noise [6], [14], can be described.

Before calculating the phase noise of $LC$-oscillators using the spectral noise analysis method, we will derive the Fourier series for the time-varying small-signal $gm$-cell gain and the $gm$-cell switching characteristic for a bipolar $LC$-oscillator. The results obtained are general and apply to CMOS $LC$-oscillators as well.

### III. Bipolar $LC$-Oscillators Time-Varying Transfer Functions

The voltage-controlled oscillator shown in Fig. 2 is used for the phase-noise analysis of bipolar $LC$-oscillators. It consists of a resonant $LC$ tank, a capacitive voltage divider ($C_A, C_B$), and a cross-coupled transconductance amplifier ($Q_1, Q_2$). The bias current source provides current $I_{TAIL}$.

This VCO is a second-order (negative resistance) oscillator in a positive feedback configuration between the resonant $LC$-tank and the transconductor via the capacitive divider. The oscillation condition is satisfied when the $LC$-tank loss conductance is compensated by the equivalent negative small-signal transconductance of the transconductor amplifier.

The relationships between the parameters of the oscillator are summarized by (6)–(8). $L$ is the tank inductance, $C$ the tank capacitance, $G_{TK} (1/R_{TK})$ the effective tank conductance, $R_L$ and $R_C$ the series loss resistances of the resonator inductor and capacitor, respectively, $n$ the capacitive divider ratio, $gm$ the transconductance of bipolar transistors $Q_1, Q_2$, $C_g$ their base-emitter capacitance, $k$ the small-signal loop gain, $V_T$ the thermal voltage, and $\omega_0$ the oscillation angular frequency [15]

$$G_{TK} = \frac{R_L}{(\omega_0 L)^2} + R_C(\omega_0 C)^2$$  (6)
Fig. 2. Bipolar LC-VCO and its main noise sources (bias not completed).

The double-sided noise power densities of the oscillator noise sources shown in Fig. 2 are given by (9)–(12). These are the tank-conductance current noise (symbol $i_{GTK}$) and rms value $i_{N}(G_{TK})$, the base-resistance ($r_{B}$) thermal noise (symbol $v_{B}$ and rms value $v_{N}(r_{B})$), the collector-current shot noise (symbol $i_{C}$ and rms value $i_{N}(I_{C})$), and the base-current shot noise (symbol $i_{B}$ and rms value $i_{N}(I_{B})$), and the equivalent output current noise (symbol $i_{BCS}$ and rms value $i_{N}(I_{BCS})$) of the current source transistor $Q_{CS}$.

\[
\begin{align*}
\frac{i_{N}^{2}(G_{TK})}{2K_{T}G_{TK}} & = \frac{2K_{T}g_{m}}{2} \\
\frac{i_{N}^{2}(r_{B})}{2K_{T}r_{B}} & = \frac{2K_{T}g_{m}}{2} \\
\frac{i_{N}^{2}(I_{C})}{2K_{T}r_{B}} & = \frac{2K_{T}g_{m}}{2} \\
\frac{i_{N}^{2}(I_{BCS})}{2K_{T}r_{B}} & = \frac{2K_{T}g_{m}}{2} \left[ 1 + 2r_{B,CS}g_{m,CS} \right]. 
\end{align*}
\]

$I_{C}$ and $I_{B}$ stand for the collector and base currents and $\beta$ for the current gain factor of the $g_{m}$-cell transistors, $g_{m,CS}$ is the transconductance and $r_{B,CS}$ is the base resistance of transistor $Q_{CS}$.

A. Time-Varying Small-Signal $g_{m}$-Cell Gain

The nonlinear voltage-to-current transfer function (referred to the LC-tank) of the transconductor, its equivalent time-varying small-signal transconductance in the presence of a large drive signal, and the (large-signal) current through the resonator are shown in Fig. 3(a)–(c), respectively [14]. Two forms of otherwise hyperbolic-tangent $V$-$I$ $g_{m}$-cell characteristic are used: a linear (dashed line) and a quadratic (solid line) approximation shown in Fig. 3(a), facilitating the description of the harmonic components of the waveforms in Fig. 3(b) and (c).

In this section, we will first compare the two $V$-$I$ characteristics shown in Fig. 3(a) with the hyperbolic-tangent ($\tanh$) characteristic of the transconductor [see (13)] [6], and then use the one that describes the $\tanh$ function more closely in the remainder of this paper.

The time-varying gain of the $g_{m}$-cell in the presence of a large drive signal and (c) (large signal) current through the resonator.

\[
\begin{align*}
\frac{V_{S,B}}{2} & = \frac{i_{OUT}(t)}{2} \tanh \left( \frac{V_{S,B}}{2V_{T}} \right). 
\end{align*}
\]

$v_{S,B}$ is the voltage swing of the oscillation signal $v = v_{S,B} \sin \omega d$ across the bases of the transconductor devices $Q_{1}$, $Q_{2}$, shown in Fig. 2.

In Fig. 3(b) are shown two approximate forms of the transconductor small-signal gain in the presence of a large oscillation signal: a square-wave (dashed line) and a triangular-wave (solid line) gain function.

As long as the oscillation signal (v) is not clipped, the gain of the accompanying noise signal has a constant value, $g$, for a square-wave function, and a linearly decaying value, starting from $g$, for a triangular-wave function. When limiting occurs, the small-signal gain reduces to zero. If the period of the oscillation signal is $1/f_{0}$, the period of the small-signal time-varying gain ($g_{N}$) is $1/(2f_{0})$. Considering the transformation from the bases to the collectors of the transconductor $Q_{1}$, $Q_{2}$, the peak value of the small-signal gain $g$ is $g_{m}/2$.

Let us now determine the average value and normalized power of the time-varying gain functions shown in Fig. 3(b), and compare them with the average value and the normalized power value of the time-varying gain originating from the $\tanh$ transconductor characteristic, given by (13).

If $d$ is the duty cycle and $\tau$ the width of the square-wave function $g_{m}(t)$ [see Fig. 3(b)], then the average values of the square-wave gain, $g_{m,0}$, the triangular-wave gain, $g_{\tau,0}$, and the gain of the $\tanh$ $g_{m}$-cell characteristic, $g_{\tau,0}$, are given by (14)–(16), respectively. ($T_2 = 0.5/f_0$)

\[
\begin{align*}
g_{m,0} & = \frac{1}{T_2} \int_{-T_2/2}^{T_2/2} \frac{\tau}{2} \frac{d\tau}{T_2} = g \frac{\tau}{2} = g d \\
g_{\tau,0} & = \frac{1}{T_2} \int_{-T_2/2}^{T_2/2} \frac{\tau}{2} \frac{d\tau}{T_2} = \frac{\tau}{2} g \frac{1 - \frac{\tau}{2}}{\tau} = g d \\
g_{N,0} & = \frac{1}{T_2} \int_{-T_2/2}^{T_2/2} \frac{4f_T}{\pi v_{S}} \frac{d\tau}{v_{S}} = \frac{I_{\text{TAL}}}{\pi v_{S,B}}. 
\end{align*}
\]
Equation (16) is solved by exchanging the time integration variable with the oscillation signal \( v = v_{S,B} \sin \omega t \), and assuming \( v_{S,B} \gg 4V_T \), i.e., the small-signal loop gain is much greater than 1.

In order to compare (14)–(16), let us first determine the duty cycle \( d \) of the dashed gain function, shown in Fig. 3(b). We also calculate the voltage swing across the resonator (and product \( gd \)), needed for the comparison of (14)–(16), and later for the derivation of the phase noise, according to (5).

If \( \pm 2V_T \) is the linear region of the piece-wise transconductor approximation [dashed V-to-I function of Fig. 3(a)], the duty cycle of the square-wave time-varying gain can be expressed as

\[
d = \frac{2}{\pi} \arcsin \left( \frac{2V_T}{v_{S,B}} \right) .
\]

The voltage swing across the LC-tank is defined as the product of the tank resistance \( 1/G_{TK} \) and the first Fourier coefficient of the current \( I_1 \) through the tank, \( v_S = I_1 R_{TK} \).

For a large value of the oscillator small-signal loop gain \( k \), output current \( i_{OUT} \) is described by a close-to-square wave with amplitude \( I_{TAIL}/2 \) [see Fig. 3(c)]. Component \( I_1 \) now equals the first harmonic of the current signal. With the aid of (7), the voltage swing across the resonator \( v_S \) then reads

\[
v_S = \frac{2}{\pi} I_{TAIL} R_{TK} = \frac{8}{\pi} k n V_T = n v_{S,B}.
\]

Assuming a 100 mV (\( \pm 2V_T \)) bipolar transconductor linear region (dashed line of Fig. 3(a), [14]) and a large small-signal loop-gain value, the duty cycle \( d \) of the square-wave gain function can be approximated as

\[
d = \frac{4V_T}{\pi v_{S,B}}
\]

or with the aid of (18) as

\[
d = \frac{1}{2k}.
\]

This expression describes the relationship between the small-signal loop gain and duty cycle for \( k \gg 1 \) (i.e., large oscillation signal generated and hard switching of the transconductor). The product \( gd \) now becomes [see (14)–(19)]

\[
\frac{gd}{I_{TAIL}} = \frac{4V_T}{\pi v_{S,B}} = \frac{I_{TAIL}}{\pi v_{S,B}}
\]

and the equality of the average values of (14)–(16) is due. The duty cycle \( d \) of the square-wave gain function shown in Fig. 3(b) can be now considered as the effective duty cycle of the triangular-wave gain function. Thus, both V-to-I approximation functions of Fig. 3(a) describe accurately the transconductor characteristic, when referring to their average values.

Let us now calculate the average normalized power of the three time-varying gain functions, viz., the square-wave power, \( P_T \), the triangular-wave power, \( P_T \), and the power of the periodic function originating from the \( \tanh \) \( g_m \)-cell characteristic, \( P_{th} \). With the aid of Parseval’s theorem [12], these are calculated as given by (22)–(24)

\[
P_T = \frac{1}{T_2} \int_{-T_2/2}^{T_2/2} g^2(t) dt = \frac{2}{T_2} \int_0^{\tau/2} g^2 dt = g^2 d = g \frac{I_{TAIL}}{v_{S,B}}
\]

\[
P_T = \frac{1}{T_2} \int_{-T_2/2}^{T_2/2} g^2(t) dt = \frac{2}{T_2} \int_0^\tau g^2 \left( 1 - \frac{t}{\tau} \right)^2 dt = \frac{2}{3} g^2 d = \frac{2}{3} P_T
\]

\[
P_{th} = \frac{1}{T_2} \int_{-T_2/2}^{T_2/2} g^2(t) dt = \frac{2}{\pi v_{S,B}} \int_0^{4V_T} \left( \frac{\partial i_{OUT}(v)}{\partial v} \right)^2 dv
\]

\[
= \frac{2g^2}{\pi v_{S,B}} \int_0^{4V_T} dv = \frac{2g^2}{\pi v_{S,B}} \left[ \frac{2}{3} V_T \frac{v}{2V_T} + \frac{4}{3} V_T \frac{v}{2V_T} \right]_0^{4V_T}
\]

\[
\approx 2.18 \frac{2g^2}{\pi v_{S,B}} V_T = 2.18 \frac{g I_{TAIL}}{\pi v_{S,B}} = 2.18 \frac{P_T}{3}
\]

As suggested by the above results, the normalized power of the triangular description of the periodic gain function, (23), is almost identical to (24) that is obtained from the transconductor characteristic of (13). The square-wave approximation of the small-signal \( g_m \)-cell gain however overestimates the normalized power of the periodic gain function obtained from the \( \tanh \) characteristic of (13) by a factor close to 3/2.

The equality of the average values, (15) and (16), and the close proximity of the normalized power values, (23) and (24), of the triangular approximation of the small-signal \( g_m \)-cell gain and the actual \( g_m \)-cell gain qualifies the former gain function of the transconductor for the oscillator analysis. Therefore, we will refer to the triangular-wave \( g_m \)-cell gain as to the \( g_m \)-cell gain in this paper. Also, we will refer to the duty cycle \( d \) of the square-wave gain function as to the duty cycle of the \( g_m \)-cell gain function.

In the remainder of this section, we will first determine the Fourier coefficients of the triangular small-signal \( g_m \)-cell gain. Then, the harmonic components of the large-signal \( g_m \)-cell transfer function will be determined. The harmonic components of the corresponding gain functions are needed for the spectral noise analysis and calculation of the phase noise, as will be demonstrated in the following sections.

B. Complex Fourier Coefficients of the Time-Varying Small-Signal \( g_m \)-Cell Gain

The noise from the LC-tank and transistors \( Q_1 \) and \( Q_2 \) is modulated by the time-varying gain \( g_{m,N} \). Consequently, noise folding occurs, i.e., noise from a number of frequencies is converted into phase-noise at one frequency. In order to calculate the noise contributions of the \( g_m \)-cell and LC-tank to the resonator phase-modulating noise component (see Fig. 1)
using spectral analysis method, the spectrum of the small-signal \( g_m \)-cell gain (i.e., spectral magnitude components) shown in Fig. 3(b) has to be determined.

The magnitudes of the harmonic components of the complex Fourier development of the time-varying gain \( g_{IN} \) are given by (25) \[ g_{2i} = gd \left( \frac{\sin((i \pi d))}{i \pi d} \right)^2. \] (25)

Gain \( g_{IN} \) has harmonic components at even multiples of the oscillation frequency only (multiples of \( 2f_0 \)). For the sake of simplicity, all coefficients are used positive (as magnitudes) in the spectral representations, as suggested by definitions (1), (2).

For \( k \gg 1 \) and with the aid of (20), (25) simplifies to

\[ g_{2i} = gd = \frac{g_m}{2} d \] (26)

for the components up to around a first zero-crossing frequency of the envelope sinc function of (25) (i.e., for \( id < 1, i = 0, 1, 2, \ldots \)). For higher frequencies, the magnitudes of the harmonic components reduce according to (25).

C. Complex Fourier Coefficients of the \( g_m \)-Cell Switching Function

The noise of the bias current source is switched between each half of the \( LC \)-tank \((\pm 1 \) squarewave function) by the \( f_0 \)-periodic transconductor limiting characteristic, shown in Fig. 3(c). This results in folding of the bias current source noise.

The complex Fourier coefficients of the square-wave switch function are positioned at odd multiples of the oscillation frequency. Their magnitudes (thus positive real numbers) are defined by (27) for \( k \gg 1 \)

\[ c_{2i+1} = \left| \sin \left( \frac{(2i+1) \pi}{2} \right) \left( \frac{2i+1}{\pi} \right) \right|. \] (27)

IV. PHASE-NOISE MODEL OF BIPOLAR LC-OSCILLATORS

For the bipolar \( LC \)-oscillator shown in Fig. 2 and its model in Fig. 1(a), we consider the case where the \( g_m \)-cell operates for a large portion of the oscillation period in the limiting region, provided a large voltage swing and small-signal loop gain. This is a typical region of operation for high-performance oscillators [15].

The noise from the \( LC \)-tank and transistors \( Q_1 \) and \( Q_2 \) (both contributions) is modulated by the small-signal time-varying \( g_m \)-cell gain. It is converted from around a number of multiples of the oscillation frequency into the phase-noise at one frequency. In a similar manner, the noise of the bias current source is periodically switched to the \( LC \)-tank via transistors \( Q_1 \) and \( Q_2 \) resulting in folding of the bias current source noise from around a number of \( f_0 \) multiples into the phase noise at around \( f_0 \).

In the following subsections, we derive a closed-form expression for the phase noise of switching bipolar \( LC \) oscillators, analyzing the oscillator noise performance in the spectral domain. After determining the oscillation condition, we evaluate for each of the noise sources (\( LC \)-tank, \( g_m \)-cell, and bias current source noise):

(a) the double-sided folded noise components at \( \pm f_0 \pm \Delta \);
(b) the single-sided phase-modulating noise component at \( f_0 + \Delta \);
(c) the phase-related noise power;
(d) the transfer function to the resonator.

Finally, we determine the phase noise of the oscillator from (a)–(d). The results obtained for bipolar oscillators are general and can be used to derive the phase-noise model of CMOS \( LC \)-oscillators.

A. Oscillation Condition

The amplitude of the fundamental current at the output of the \( g_m \)-cell is obtained by multiplying the oscillation signal voltage components \( (v_{SB}/2) \) by the \( g_m \)-cell harmonics: \( v_{SB}/2 \) at \( \pm f_0 \) multiplied by \( g_0 \) and \( g_{2i}/g_2 \) and shifted to \( f_0 \). After transformation to positive frequencies, the obtained current amplitude is converted into voltage at the output of the resonator by its impedance at resonance \( R_{TK} \).

\[ \frac{v_{SB}(-f_0)}{2}g_0 + \frac{v_{SB}(f_0)}{2}g_0 + \frac{v_{SB}(-f_0)}{2}g_2 + \frac{v_{SB}(f_0)}{2}g_2 \]

\[ R_{TK} = (g_0 + g_2)v_{SB}R_{TK}. \] (28)

As the signal amplitudes at the input of the \( g_m \)-cell and the output of a divider preceding the resonator are equal in steady state, the oscillation condition for the switching oscillators can be derived from (28) as

\[ g_0 + g_2 = nG_{TK} \] (29)

or with the aid of (25) as

\[ gd \left( 1 + \left( \frac{\sin(\pi d)}{\pi d} \right)^2 \right) = 2gd = gm = nG_{TK}. \] (30)

The duty cycle \( d \) determined from (30) results in (20), as expected. The product \( gd \) equals \( nG_{TK}/2 \), being the average value of the \( g_m \)-cell time-varying gain functions given by (14)–(16).

B. \( LC \)-Tank Noise

Harmonic components of the \textit{band-limited} \( LC \)-tank noise \( (v_N) \) at input of the \( g_m \)-cell (i.e., output of the divider) at \( \pm f_0 \pm \Delta \) and harmonic components \( g_0 \) and \( g_{2i} \) of the time-varying small-signal \( g_m \)-cell gain are shown in Fig. 4, together with spectral representations of the convolution (i.e., multiplication and shifting in the frequency domain) between these harmonic components.

(a) After convolution, the \( LC \)-tank noise current components at the output of the \( g_m \)-cell at \( \pm f_0 \pm \Delta \) are given by (31) and (32), as shown in Fig. 4.

\[ i_{NO}(f_0 \pm \Delta) = g_nv_N(f_0 \pm \Delta) + g_{2i}v_N((-f_0 \pm \Delta)) \] (31)

\[ i_{NO}(-f_0 \pm \Delta) = g_nv_N(-f_0 \pm \Delta) + g_{2i}v_N(f_0 \pm \Delta). \] (32)
The phase-modulating noise component (single-sided) $i_{PM}$ at frequency $f_0 + \Delta$ is obtained by combining the current noise contributions $i_{NO}$ at $\pm f_0 \pm \Delta$ as implied by the generic phase noise model and (1)-(3)

$$i_{PM}(f_0 + \Delta) = \frac{1}{2}i_{NO}(f_0 + \Delta) + \frac{1}{2}i_{NO}(-f_0 + \Delta)$$

$$+ \frac{1}{2}i_{NO}(-f_0 - \Delta) + \frac{1}{2}i_{NO}(f_0 - \Delta). \quad (33)$$

After summing the currents of the correlated noise components ($g_{\pm 2}\nu N(f_0 + \Delta)$ correlated with $g_{\pm 2}\nu N(-f_0 - \Delta)$ and $g_{\pm 2}\nu N(f_0 - \Delta)$ correlated with $g_{\pm 2}\nu N(-f_0 + \Delta)$) as

$$i_{PM}(f_0 + \Delta) = (g_0 + g_2)\nu N(f_0 + \Delta) + (g_0 + g_2)\nu N(f_0 - \Delta) \quad \text{(34)}$$

the phase-related noise power at $f_0 + \Delta$ is calculated

$$i_{PM}^2(f_0 + \Delta) = \frac{1}{2}[(g_0 + g_2)^2\nu N^2(f_0 + \Delta)$$

$$(g_0 + g_2)^2\nu N^2(f_0 - \Delta)] \quad (35)$$

where the factor $1/2$ relates to the rms power.

(c) As the magnitudes of the harmonic components of the LC-tank white noise at input of the transconductor are equal ($\nu N^2(R_{TRK}) = \nu N^2(R_{TRK})$, (35) reduces to

$$i_{PM}^2(R_{TRK}) = \frac{(g_0 + g_2)^2\nu N^2(R_{TRK})}{n^2}. \quad (36)$$

(d) The gain from the double-sided LC-tank voltage noise density $\nu N^2(R_{TRK}) = 2KTR_{TRK}$ back to the LC-tank current-noise density (via the divider and $g_{m}$-cell) is obtained from (36) as

$$g^2(R_{TRK}) = \frac{(g_0 + g_2)^2}{n^2}. \quad (37)$$

With the aid of (4) and (29), the contribution of the LC-tank noise to the phase noise, prior to shaping by the LC-tank impedance $Z(\Delta u)$, equals

$$L(R_{TRK}) = \frac{i_{PM}^2(R_{TRK})}{\nu N^2/R_{TRK}} = \frac{2(g_0 + g_2)^2\nu N^2(R_{TRK})}{n^2\nu N^2}$$

$$= \frac{2G_{TRK}^2\nu N^2(R_{TRK})}{n^2\nu N^2} = \frac{4KTR_{TRK}}{n^2\nu N^2}. \quad (38)$$

If we relate the contribution of each noise source $\nu N$ (prior to shaping by the LC-tank band-pass characteristic) to phase noise $L(\nu N)$ by a noise factor $F(\nu N) \Rightarrow \nu N^2L(\nu N)/(4KTR_{TRK})$, we obtain the LC-tank noise factor $F(R_{TRK})$ from (38) as

$$F(R_{TRK}) = \frac{\nu N^2L(R_{TRK})}{4KTR_{TRK}} = 1. \quad (39)$$

The contributions of other oscillator noise source to phase noise can be conveniently compared to the contribution of the LC-tank by referring to its noise factor of one.

C. Base-Resistance Thermal Noise

The base-resistance thermal noise $\nu N(\nu B)$ at $\pm(2i+1)f_0 \pm \Delta$ fold to the resonator via the harmonic components $g_{2i+2}$, given the small-signal time-varying gain shown in Fig. 3(b). The maximum gain is $g = g_m/2$ in the linear and zero in the $g_m$-cell limiting region, suggesting that $\nu N(\nu B)$ does not contribute to the phase-noise in the limiting region.

(a)–(b) Harmonic components of the base-resistance noise at $\pm f_0 \pm \Delta$ convolve in the same wave as the LC-tank noise at these frequencies, which is shown in Fig. 4(a) and (b). Unlike the band-limited LC-tank noise, the $g_m$-cell noise folds back to the resonator around the oscillation frequency from around all odd harmonic components.

Assuming all base-resistance noise components undergo the same transformation mechanism, the convolution of the noise components at $\pm f_0 \pm \Delta, \pm(2i+1)f_0 \pm \Delta$ with $g_0, g_2, \ldots, g_{2i}$ results in a PM noise component at $f_0 + \Delta$, which is obtained by generalizing (34) as

$$i_{PM}(f_0 + \Delta) = (g_0 + g_2)\nu N(f_0 + \Delta)$$

$$+ (g_0 + g_2)\nu N(f_0 - \Delta)$$

$$+ (g_0 + g_4)\nu N(3f_0 + \Delta)$$

$$+ (g_2 + g_4)\nu N(3f_0 - \Delta) + \cdots$$

$$+ (g_{2i-2} + g_{2i})\nu N((2i-1)f_0 + \Delta)$$

$$+ (g_{2i-2} + g_{2i})\nu N((2i-1)f_0 - \Delta). \quad (40)$$

For example, noise components at $\pm 3f_0 \pm \Delta$ convolve with $g_{\pm 2}$ and $g_{\pm 4}$, resulting in the contribution shown in the third and fourth rows of (40).

(c) As the magnitudes of the harmonic components of the white base-resistance noise are equal, the phase-related noise power at $f_0 + \Delta$ is obtained from (40) as

$$i_{PM}^2(\nu B) = [(g_0 + g_2)^2 + (g_0 + g_4)^2 + \cdots] \nu N^2(\nu B). \quad (41)$$
Assuming equality of the neighboring coefficients $g_{2i}$, and by replacing $2g_{2i} - 2g_{2i+1}$ by $g_{2i}^2 + g_{2i+1}^2$, subsequently, the above summation becomes

$$i_{PM}^2(r_B) = \left[2g_{2i}^2 + 4g_{2i}^2 + 4g_{2i+1}^2 + \ldots + 4g_{2i}^2 + g_{2i+1}^2 \right] v_N^2(r_B)$$

$$= 2 \left( g_{2i}^2 + 2 \sum_{i=1}^{\infty} |g_{2i}|^2 \right) v_N^2(r_B)$$

$$= v_N^2(r_B) \frac{1}{T_2} \int_{-T_2/2}^{T_2/2} g_{2i}^2(t) dt.$$  \hspace{1cm} (42)

The summation obtained can be determined employing the Parseval’s theorem, by relating this summation power series to (23) [12]. The phase-related noise power of the base-resistance $g_{m}$-cell noise at $f_0 + \Delta$ is now obtained from (22), (23), and (42). This is given by (43)

$$i_{PM}^2(r_B) = \frac{4}{3} g_{2i}^2 d v_N^2(r_B).$$  \hspace{1cm} (43)

Note that in the above analysis infinite bandwidth of operation of the transconductor devices and their noise sources have been implicitly assumed. Therefore, (43) is the worst-case noise contribution of the $g_{m}$-cell base-resistance noise. Namely, the summation of (42) is limited by the finite bandwidth of the $g_{m}$-cell devices and corresponding noise source. If the bandwidth were limited to a value $f_B$, the number of the folding components in (42) would be $f_B/(2f_0)$. For example, if $f_B$ is the zero crossing of the sinc envelope function of (25), the number of the folding components is $2f_0/(2f_0d) = 2k$ [see Fig. 3(b)]. We will return to this point later.

(d) The gain from the double-sided base-resistance voltage noise density $v_N^2(r_B)$ to the $LC$-tank as current-noise density, via the $g_{m}$-cell, is now derived from (43) as

$$g^2(r_B) = \frac{4}{3} g_{2i}^2 \left( \frac{g_{m}}{2} \right)^2 = \frac{4}{3} \frac{1}{2k} (nkG_{TK})^2 = \frac{2}{3} kn^2G_{TK}^2.$$  \hspace{1cm} (44)

The gain from the base-resistance noise is a function of the duty cycle $d$ and the maximum of the small-signal $g_{m}$-cell gain $g$ as well as a function of the small-signal loop gain $k$, divider ratio $n$, and $LC$-tank conductance $G_{TK}$.

The contribution of the $v_N^2(r_B)$ to the phase noise can be now calculated from (43) and (44). This is given by (45)

$$\mathcal{L}(r_B) = \frac{i_{PM}^2(r_B)}{v_N^2} \frac{1}{2} = \frac{4}{3} \frac{1}{2k} \left( \frac{kG_{TK}}{3v_0^2} \right)^2 = \frac{8}{3} \frac{kG_{TK}G_{TB}G_{TK}k}{3v_0^2}.$$  \hspace{1cm} (45)

Taking into account both transistors of the $g_{m}$-cell, the total base-resistance phase-noise contribution is twice that of (45), resulting in a noise factor $F(2r_B)$. (46)

$$F(2r_B) = \frac{\mathcal{L}(2r_B)}{\mathcal{L}(r_B)} = \frac{8}{12} \frac{kG_{TK}G_{TB}G_{TK}2nk}{3v_0^2}$$

$$= \frac{4}{3} \frac{kG_{TB}G_{TK}}{2nk},$$  \hspace{1cm} (46)

where $c = r_B2nG_{TK}$ is a start-up constant.

This result suggests that the contribution of the $g_{m}$-cell base-resistance noise to the phase noise of the oscillator under consideration is directly proportional to the small-signal loop gain $k$ (and thus to the power consumption) and parameters $n$ and $c$, the latter relating $r_B$ of the transconductor transistors and the quality of the resonator.

**D. Transconductor Shot Noise**

The contribution of the transconductor shot noise to the PM noise component at the resonator can be determined following the procedure outlined in the previous section. As collector-current and base-current shot noise sources are already at input and output of the resonator, respectively, their phase-noise contributions can be determined by taking into account the effect of the noise modulation due to the switching of the transconductor. Even though the $g_{m}$-cell base-current shot noise contributes to the $LC$-tank phase-modulating component at all times, it is around an order of magnitude smaller than the collector-current shot noise [see (11)] as well as its contribution to the phase noise. We will therefore consider the collector-current shot noise only in the remainder of this section. However, a special care has to be taken in the design of the $g_{m}$-cell base-bias circuitry, as otherwise the contribution of the base-current shot noise may be enhanced (e.g., for a low-impedance of the $g_{m}$-cell base biasing circuit).

When only $Q_1$ (or $Q_2$) is active, $i_N(I_C)$ does not flow to the resonator, due to the large impedance at the emitter of the $g_{m}$-cell transistor. This is expected, considering $Q_1$ as a common-base like transistor in this case. The situation where only $Q_4$ is active is described in Fig. 5(a); the collector-emitter oriented noise source $i_C$ is split into the collector-to-ground.
and ground-to-emitter oriented sources, \(i_C^e\) and \(i_C^c\), respectively. After \(i_C^e\) is referred from emitter to collector of the common-base-like transistor \(Q_1\), thus with a gain of around one [see Fig. 5(b)], the noise sources \(i_C^e\) and \(i_C^c\) cancel at the resonator, Fig. 5(c).

The transconductor collector-current shot noise contributes to the PM noise component at the resonator when \(Q_1\) and \(Q_2\) are simultaneously active. We will first determine the contribution of \(i_N(I_C)\) to the LC-tank in the linear region of the transconductor using circuit analysis, and then consider the modulation of the transconductance by referring to the \(g_{m}\)-cell small-signal time-varying transfer function.

Each current noise source can be split as shown in Fig. 6(a): the collector-emitter oriented noise sources \(i_C\) are split into the collector-to-ground and ground-to-emitter oriented sources, \(i_C^j\) and \(i_C^e\), respectively (indices 1 and 2 refer to \(Q_1\) and \(Q_2\) in Fig. 6). Whereas the common-mode sources \(i_C^m\) can be neglected, the sources \(i_C^j\) appear across a half the tank resistance \(1/2R_{TK}\) each [see Fig. 6(b)], generating the single-sided PM noise voltage component as given by (47). Here, we have assumed that noise \(i_C\) adds completely to the phase noise (with single-sided noise current density \(2R_N(I_C)\))

\[
v_{PM}^2(I_C) = \frac{2R_N(I_C)I_{TD}^2}{4}.
\]

The total contribution of each collector-current shot-noise source in the \(g_{m}\)-cell linear region to the phase-modulating noise current component, referred to the complete LC-tank \(R_{TK}\), equals

\[
\frac{v_{PM}^2(I_C)}{R_{TK}^2} = \frac{2R_N(I_C)}{2} = KT\frac{g_m}{2}.
\]

This shows that the total gain from the \(g_{m}\)-cell collector-current shot noise sources to the LC-tank equals \(1/2\) [compare (11) and (48)], when both \(g_{m}\)-cell transistors are simultaneously active.

(a–c) The LC-tank referred power of the \(g_{m}\)-cell shot-noise \(KTg_m/2\) is modulated by the switching function of the \(g_{m}\)-cell, that is, by the function \(g_{IN}\) of Fig. 3(b), however with a gain of 1. The total power of the phase-modulating noise component at \(f_0 + \Delta\) originating from each \(g_{m}\)-cell collector-current shot noise source, is obtained by averaging the transistor transconductance (and noise density of (48), accordingly) over a period of oscillation. Calculating the average of the time-varying noise modulating function (Fig. 3(b) with \(g = 1\)) from (15) as \(d\), \(\frac{v_{PM}^2(I_C)}{2}\) becomes

\[
\frac{v_{PM}^2(I_C)}{2} = \frac{2R_N(I_C)}{2}.
\]

(d) The gain from the double-sided collector-current shot-noise to the PM noise component referred to the LC-tank now equals

\[
g^2(I_C) = \frac{d}{2} = \frac{1}{4k}.
\]

being inversely proportional to the small-signal loop gain.

The contribution of the \(i_N(I_C)\) to the phase noise becomes

\[
L(I_C) = \frac{v_{PM}^2(I_C)}{v_S^2/2} = \frac{2R_N(I_C)}{v_S^2} = \frac{2KTN_c}{v_S^2} = \frac{KTN_c}{v_S^2}.
\]

Taking into account the contributions of the collector-current shot-noise sources from both transconductor transistors, a noise factor \(F(2I_C)\) is obtained as given by (52)

\[
F(2I_C) = \frac{v_{PM}^2(I_C)}{4KTN_c} = \frac{2KTN_c}{4KTN_c} = \frac{n}{2}.
\]

Equation (52) suggests that the phase-noise contribution of the \(g_{m}\)-cell collector-current shot-noise is independent of the small-signal loop gain and power consumption, but proportional to the capacitive divider ratio \(n\), for \(k \gg 1\).

E. Bias Current Source Noise

Folding of the bias current source (BCS) noise is a result of the operation of the \(g_{m}\)-cell in the limiting region. Having the harmonic components at odd multiples of the oscillation frequency, \(c_{2(2i+1)}\), given by (27), the \(g_{m}\)-cell switching function [see Fig. 3(c)] converts the BCS noise components at around even multiples of the oscillation frequency back to the LC-tank at around the oscillation frequency. Note that we refer to the magnitudes of the Fourier components of the transconductor switching characteristic, thus, positive real numbers. This is done in order to comply with the definition of the phase noise given by Fig. 1(b) and (1)–(4).

For the sake of simplicity, we will first consider the convolution between the harmonic components \(c_{2i+1}\) at \(\pm f_0\) and \(\pm f_0\) with the BCS noise components at \(\pm 2f_0\) and \(\pm 2f_0\). Then, we will derive an expression for the contribution of the BCS noise to the phase noise, by generalizing this result for all harmonic products.

(a) The convolution of the BCS noise \(i_{B}(I_{BCS})\) at \(\pm f_0\) with harmonics \(c_{\pm l}\) results in a differential noise component \(i_{N,C}\), referred to the output of the transconductor at \(\pm f_0\), that is given as follows:

\[
i_{N,C}(f_0 \pm \Delta) = c_{-1}i_{N}(2f_0 \pm \Delta) + c_{1}i_{N}(\pm \Delta).
\]

(b) The noise component at \(\pm 2f_0\), that is given as follows:

\[
i_{N,D}(-f_0 \pm \Delta) = c_{-1}i_{N}(\pm \Delta) + c_{1}i_{N}(\mp f_0 \pm \Delta).
\]
Similarly, a convolution of the noise $i_N(I_{BCS})$ at $\pm 2f_0 \pm \Delta$ and $\pm 4f_0 \pm \Delta$ with harmonics $c_{\pm \Delta}$ results in

$$i_{N,0}(f_0 \pm \Delta) = c_{-\Delta}i_N(4f_0 \pm \Delta) + c_{\Delta}i_N(-2f_0 \pm \Delta),$$  
(55)

$$i_{N,0}(-f_0 \pm \Delta) = c_{-\Delta}i_N(2f_0 \pm \Delta) + c_{\Delta}i_N(-4f_0 \pm \Delta).$$  
(56)

(b) A differential phase-modulating noise component (single-sided spectrum) at a frequency $f_0 \pm \Delta$ is obtained by combining the current noise contributions $i_{N,0}$ at $\pm f_0 \pm \Delta$ as given by (1), (2). The noise components at frequencies $\pm \Delta$ (i.e., $i_N(\pm \Delta)$) contribute purely to an AM component at $f_0 \pm \Delta$ [6], [16], [17]. Nonetheless, the noise components at $\pm \Delta$ (e.g., $1/f$ noise) can be converted to a frequency-modulating (FM) component, via AM-to-FM conversion mechanism of the LC-tank varactor, which can’t be distinguished from PM components [16], [17]. However, as this is beyond the scope of this paper, we will proceed our analysis accounting for PM-contributing components only. Doing so, (53)–(56) yield (57) (BCS noise at $\pm 2f_0 \pm \Delta$ considered)

$$i_{PM,\text{DIFF}}(f_0 + \Delta) = (c_1 + c_3) i_N(2f_0 + \Delta) + (c_1 + c_3) i_N(2f_0 - \Delta) + \ldots.$$  
(57)

(c) From (57), a general rule can be found for the convolution between the harmonics of the BCS noise and the transistor switching characteristic of Fig. 3(c). Assuming the equalities $c_{(2\nu+1)} = c_{(2\nu+1)}$, and $i_N(\pm \Delta) = i_N(\pm 2f_0 \pm \Delta) = i_N(I_{BCS})$, a complete formula for the differential phase-related power of the BCS noise referred to the LC-tank is given by (58)

$$i_{PM,\text{DIFF}}(I_{BCS}) = \left((c_1 + c_3)^2 + (c_1 + c_3)^2 + \ldots \right) / 2.$$  
(58)

Using the weights of a 50% duty cycle square-wave [12], the summation series of (58) can be calculated as $i_{PM,\text{DIFF}}(I_{BCS}) = i_N^2(I_{BCS})$. As the current through the LC-tank is one half the differential current component, the phase-related noise power (related to the resonator as a whole) equals

$$i_{PM,\text{DIFF}}(I_{BCS}) = \frac{i_{PM,\text{DIFF}}(I_{BCS})}{4} = \frac{2}{4}.$$  
(59)

(d) The transfer function from the double-sided bias current noise density $i_N^2(I_{BCS})$ to the LC-tank is now derived from (59) as $g^2(I_{BCS}) = 1/4$.

The contribution of the bias current source noise to the phase noise is obtained as

$$\mathcal{L}(I_{BCS}) = \frac{\gamma^2(I_{BCS})}{v_S^2 / 2} = \frac{KTg_m(1 + 2r_Bg_m)}{v_S^2}$$  
(60)

where we have used $g_m = 2g_m$ and assumed $2r_B = r_B$, given approximately the same transit frequencies ($f_T$) of transistors $Q_1$, $Q_2$, and $Q_{CS}$ (thus, $Q_{CS}$ twice as large as $Q_1$, $Q_2$ is assumed).

Finally, a noise factor $F(I_{BCS})$ results, as given by (61)

$$F(I_{BCS}) = \frac{2 \gamma^2(I_{BCS})}{4Kg_m} = \frac{KTg_m(1 + 2r_Bg_m)}{4Kg_m}$$  
(61)

For transistor $Q_{CS}$ (see Fig. 2) of arbitrary dimensions, and $\alpha t_{B,C} = r_B/Q_{CS}$ larger than $Q_1$, $Q_2$ for $\alpha > 1$, and smaller for $\alpha < 1, \alpha$ being a ratio of the areas of $Q_{CS}$ and $Q_1$, $Q_2$, a general formulation of (61) reads

$$F(I_{BCS}) = F(I_{C,CS}) + F(r_{B,CS}) = k \frac{n + 2n_{kC}/\alpha}{2}.$$  
(62)

where $F(I_{C,CS})$ and $F(r_{B,CS})$ stand for the noise factors of the bias current source collector-current shot noise and base-resistance thermal noise, respectively.

Comparing the result obtained with the noise factors of the $g_m$-cell collector-current shot noise and base-resistance thermal noise sources, (46) and (52), (62) becomes

$$F(I_{BCS}) = k(F(2L) + 3F(2r_B)/\alpha).$$  
(63)

We observe from (63) that the contribution of the bias current source noise to the phase noise of the LC-oscillator under consideration is larger than all other contributions together [9]. In particular, the contribution of the BCS is a factor of around $k$ larger than the contribution of the $g_m$-cell for $\alpha \sim 3$. For a typical small-signal loop gain of around 10, this would result in an unacceptable degradation of the phase noise of around 10 dB from the calculations. Finally, it is important to note that (61) holds for $k > 1$, thus for the hard switching of the transistor. At $k = 1$, the contribution from the BCS noise is small, being a common-mode signal. Without loss of generality, and for a convenient formulation, we will use (61) in the remainder of this paper unless stated otherwise.

Finally, note that the result obtained assumes infinitely large bandwidth of the operation of the bias current source devices and their noise sources. Equation (61) therefore overestimates the contribution of the BCS noise to the phase noise, as the operational bandwidth of the devices and their noise sources is limited. Moreover, the $g_m$-cell switching function deviates from a perfect square-wave [see Fig. 3(c)], thereby reducing the contribution of the BCS noise additionally. We will come to this point soon.

F. Noise Factor and Phase Noise for Bipolar LC-Oscillators

Assumed to be uncorrelated, all noise sources, viz., the tank conductance noise, the base resistance noise, the transistor shot noise, and the bias current noise, add up to an equivalent phase-modulating noise component at the resonator. With the aid of (39), (46), (52), and (61), the noise factor of the bipolar LC oscillator now reads as given by (64) and (65)

$$F = F(R_{TK}) + F(2L) + F(2r_B) + F(I_{BCS})$$  
(64)

$$F = 1 + (1 + k) \frac{n}{2} + \left( \frac{2}{3} + k \right) n_{kC}.$$  
(65)
The phase noise at an offset frequency $\Delta$ from the oscillation frequency $f_0$ is determined with the aid of (5) as

$$
L = \frac{2\pi^2 \Phi_{PM, TOT}}{8\pi^2 (4\pi C_{TOT} \Delta)^2} = \frac{4KGTG_{TK} F}{L(R_{TK}) + L(2I_C) + L(2\tau_B) + L(I_{BCS})},
$$

(66)

where $C_{TOT}$ stands for the $\text{LC}$-tank total capacitance. Substituting (18) into (66), the phase noise finally becomes

$$
L = \frac{4KGTG_{TK}}{(4\pi C_{TOT} \Delta)^2} \left( \frac{\pi}{8V_T} \right)^2 \frac{2 + (1 + k)\frac{2}{3} + (\frac{3}{2} + k) nkc}{k^2 n^2}.
$$

(67)

The result obtained is a comprehensive description of the phase-noise phenomenon in the $\text{LC}$ oscillator under consideration. The developed model does not require use of numerical solvers, yet providing oscillator designers with a valuable tool for analysis and synthesis of high-performance oscillators. The phase-noise model of (67) is valid as long as voltage swing and phase-noise improve with current consumption (and $k$). Conditions for the maximum voltage swing and small-signal loop gain can be found in [15].

It is important to note that (65) gives the worst-case noise factor of the bipolar oscillator under consideration, thereby overestimating its phase noise by using (67). The small-signal loop-gain related contributions, viz., the base-resistance noise contribution of the transconductor, (46), and the base-resistance and collector-current noise contributions of the bias current source, (61), are calculated implicitly assuming infinite operational bandwidth of both the noise sources and the oscillator devices. However, the limited operation bandwidth of the devices and noise sources of the transconductor and bias current source alleviate the effect of the noise folding: the noise power around higher harmonic components is small as are the corresponding devices gain parameters, as expected. Moreover, a finite transconductor switching time (thus not an ideal square-wave function) additionally ameliorates the BCS noise contribution. The exact small-signal loop-gain dependent contributions are therefore smaller than (46) and (61). It is expected that low-power, low-frequency, large loop-gain oscillators with high $f_P$ of the devices available approach the results of the calculations: the lower the oscillation frequency, the more noise components fold within the operational bandwidth; the faster the switching of the $g_m$-cell, the more BCS noise components fold to the resonator. Deviations of the calculated phase-noise contributions from those of a “real” oscillator design can be estimated using oscillator simulators accounting for the bandwidth limitations. Next sections detail this.

However, the results obtained are intuitive and describe qualitatively the complex phase-noise generating mechanisms in $\text{LC}$-oscillators. The formulations derived are amenable for making qualitative and quantitative design decisions as they describe the oscillator phase-noise performance using electrical parameters.

G. Bipolar $\text{LC}$-Oscillator With Directly Coupled $\text{LC}$-Tank and $g_m$-Cell

Bipolar $\text{LC}$-oscillators are often designed with their $\text{LC}$-tank directly coupled to the active part, that is, the oscillator of Fig. 2 with a short between the bases and corresponding collectors of $Q_1$, $Q_2$ ($V_B = V_C$), or with $C_A = 0$ and $C_B = \infty$. Such an oscillator is just a special case of the oscillator of Fig. 3, with a capacitive divider ratio $n = 1$.

The phase-noise model derived for the directly coupled bipolar $\text{LC}$-oscillator is readily obtained from the model derived in the previous sections by substituting $n = 1$ in (65) and (67). This is given by (68) and (69).

$$
F = \frac{1 + \frac{k}{2} + \left(\frac{2}{3} + k\right) nkc}{k^2 n^2}.
$$

(68)

$$
L = \frac{4KGTG_{TK}}{(4\pi C_{TOT} \Delta)^2} \left( \frac{\pi}{8V_T} \right)^2 \frac{1 + \frac{1+k}{2} + \left(\frac{3}{2} + k\right) nkc}{k^2 n^2}.
$$

(69)

An $\text{LC}$ oscillator with $n = 1$ is a low-power counterpart of an oscillator with $n > 1$. A larger effective conductance of the $g_m$-cell seen by the $\text{LC}$-tank ($g_m/2$) compensates the loss $\text{LC}$-tank conductance for lower bias current ($I_{T\text{AIL}}$ in Fig. 2). The oscillator phase-noise analysis described in previous sections applies to directly coupled oscillators as well.

As CMOS $\text{LC}$ oscillators are designed with a directly coupled $\text{LC}$-tank and active part, we can conveniently use the above formulations for performance comparison between the bipolar and CMOS $\text{LC}$-oscillators (as well as derivation of the phase-noise model for CMOS $\text{LC}$-oscillators [16]–[20]).

H. Implications of the Phase-Noise Model on Designs of Bipolar $\text{LC}$-Oscillator

In this section we will discuss the implications of the results obtained from the spectral noise analysis on the synthesis of bipolar $\text{LC}$-oscillators.

The contribution of the $\text{LC}$-tank noise to the phase noise is a constant ($\text{LC}$-tank noise factor $\sim 1$), as is the contribution of the $g_m$-cell shot noise ($\sim 1/2n$). The phase-noise contribution of the $g_m$-cell base-resistance noise could surpass the contributions of the $\text{LC}$-tank noise and $g_m$-cell shot noise, given the operating small-signal loop gain ($k$) as well as the $\text{LC}$-tank designed and the $g_m$-cell transistors base resistance (factor $c$). This contribution is larger for a larger $k$. But, the voltage swing increases with $k$ as well, resulting in a better net phase noise. All these contributions are however minor compared to the phase-noise contribution of the bias current source.

To better understand design implications of the phase-noise model derived let us consider three relevant design cases. Closer inspection of (67) reveals that for a large $c$ ($\sim 1$, thus, a large $\text{LC}$-tank conductance), and $k \sim 1$, the phase noise would be proportional to a constant $c$. For a large $c$ and $k \gg 1$, the phase noise would improve with a small-signal loop gain $k$. On the other hand, for a low $c$ ($\sim 0.01$, thus, a high quality $\text{LC}$-tank), $k \gg 1$, and suppressed bias current-source noise [9], the phase noise would be proportional to $1/k^2$. Comparing the former with the latter two designs (the former is a design with a low quality $\text{LC}$-tank, unsuppressed BCS noise, and a small $k$, the second is a design with a low quality $\text{LC}$-tank, unsuppressed BCS noise, and
a large \( k \), while the latter is a design with a high quality LC-tank, suppressed BCS noise, and large \( k \), a factor of \( k \) and \( k^2 \) better phase noise results, respectively. In other words, for a typical value of \( k \approx 10 \), a factor \approx 100 (or \approx 20 \text{ dB}) and \approx 10 \text{ difference between low- and high-performance designs may result. These observations, derived from the phase-noise model obtained, are valuable to oscillator designers to make proper decisions in an early design phase.

V. VERIFICATION OF PHASE-NOISE MODEL FOR BIPOLAR LC-OSCILLATORS

The phase-noise model obtained from the spectral analysis of noise in LC-oscillators is verified in this section by comparing the theoretical calculation results of the phase-noise model for a bipolar LC-oscillator with the results obtained from the simulations. Other phase-noise analysis methods for bipolar LC-oscillators found in literature are also discussed in this section.

A. Results of Simulations for the Bipolar LC-Oscillator

In this section, calculation and simulation results for the bipolar oscillator shown in Fig. 2 are compared in terms of noise factors, \( F(2I_C) \), \( F(2r_B) \), \( F(r_{BCS}) \), \( F(I_{CCS}) \) (the latter two being the noise factors of the base-resistance thermal noise and collector-current shot noise of a bias current source transistor). The noise factors simulated are determined relative to the phase-noise contribution of the LC-tank, \( F(R_{TK}) = 1 \). The oscillator shown in Fig. 2 is simulated using SpectreRF with the following parameters (extracted from the layout [9]): oscillation frequency \( f_0 \approx 5.74 \text{ GHz} \), tank resistance \( R_{TK} \approx 340 \text{ } \Omega \), capacitive divider ratio \( n \approx 1.6 \), start-up constant \( c \approx 0.25 \), supply voltage \( V_{CC} \approx 2.2 \text{ V} \). Emitter area of the bias current source transistors is twice that of the \( g_m \)-cell transistors.

For fair comparison of the simulation and calculation results, the simulated circuit parameters have been used in the calculations. In particular, a good estimation of the small-signal loop gain \( k \) is needed, as it is a corner-stone parameter of the phase-noise model.

The small-signal loop gain \( k \) equals \( R_{TK} g_m - C_A/(2n) \), where \( g_m - C_A/(2n) \) stands for the calculated transconductance of the \( g_m \)-cell that is seen by the LC-tank. However, the effective transconductance of the cross-coupled \( g_m \)-cell seen by the LC-tank, \( g_m - EF/(2n) \), which is determined from the simulations, is smaller than \( g_m - C_A/(2n) \). The most apparent reason is the feedback effect of the transistors’ emitter resistors reducing the transconductance. A relationship between the small-signal loop gain \( k \) used in the phase-noise formulations and the effective small-signal loop gain \( k_{EF} \) (\( k > k_{EF} \)) obtained from the simulations is given by (70)

\[
k = g_m - C_A - \frac{R_{TK}}{2n} > g_m - EF - \frac{R_{TK}}{2n} = k_{EF}.
\]

A factor of around 1.5 between the two loop-gain values (\( k \) and \( k_{EF} \)) has been estimated from the simulation for \( k \gg 1 \). Results of simulations and calculations (obtained by replacing \( k \) with \( k_{EF} \) in the corresponding formulæ) are compared for the same effective \( k \), and then referred to the small-signal loop gain \( k \) used in the phase-noise derivations. Four distinct \( k \) values larger than one have been considered: 4, 6, 8, and 9.6, the latter providing the best phase-noise simulated.

The calculated noise factor of the transconductor collector-current shot noise for bipolar LC-oscillators, (52), is compared with the simulation results in Fig. 7. A very good agreement has resulted.

The calculated and simulated noise factors of the transconductor base-resistance thermal noise [see (46)], the bias current source collector-current shot noise \([nk/2 \text{ in (61)}]\), and bias current source base-resistance thermal noise \([nk^2c \text{ in (61)}]\) are compared in Figs. 8, 9(a) and (b).

As observed from the diagrams, the calculated relationships between the noise factors and small-signal loop gain match those obtained from the simulations, thereby validating the phase-noise model derived, i.e., (46), (52), (61), and (67).

As expected, the simulated small-signal loop-gain related contributions, Figs. 8, 9(a) and (b), account for around 70%–80% of the contributions obtained from the calculations for \( k \gg 1 \). As already explained in the previous section, the difference comes from the limited operational bandwidth of the devices and noise sources of the transconductor and bias.
current source as well as a finite transconductor switching time [6] accounted for in simulations (but not in calculations).

The match between the phase-noise simulated and calculated results depends on $k$, $f_0$, and operational bandwidth of the devices. The detail required to closely describe the small-signal loop-gain (and, hence, power-consumption) related contributions is adequately captured in modern computer-aided design tools.

The results of this comparative study verify the derived oscillator phase-noise model. Though overestimating the phase noise performance, the formulations derived are intuitive and give a valuable insight into the origin of phase noise in the LC oscillator under consideration. In the remainder of the paper we will refer to the formulations introduced while drawing conclusions impacting the design of LC oscillators.

**B. Existing Phase-Noise Models for Bipolar LC-Oscillators**

Numerous phase-noise analysis methods can be found in literature, among them frequency-domain, time-domain, linear, and nonlinear analyses. Linear frequency-domain analysis assumes that noise is superimposed directly onto the carrier [1]. Another way to use linear analysis in the frequency domain is to consider the noise sources to be additive to the phase of the carrier [2], [3]. Similarly, noise added to the phase [4]–[6] and carrier [7], [8] in the time domain has been used for phase-noise modeling in oscillators. Some of these methods have provided closed-form expressions for the oscillator phase noise [16], [20]–[22].

The observations made in this paper agree with the findings in [6] that provides expressions for the noise factors of the $g_m$-cell base-resistance thermal noise and the bias current source collector-current shot noise. To account for a finite bandwidth of operation of the oscillator devices and noise sources, empirical parameters are added to the corresponding noise factors in [6]. The results of this paper, however, provide an intuitive, closed-form formulation for the phase noise, while using electrical circuit parameters.

The results of the phase-noise analysis applied to CMOS LC-oscillators can be found elsewhere [5], [16]–[20], lending themselves to use in hand calculations and computer-aided design.

Note that the results in [5], [6], [12], [15]–[20] refer to hard-switching oscillators where transconductors operate for a large portion of the oscillation period in the limiting region.

**VI. Test Oscillator Design**

A test oscillator shown in Fig. 10(a) has been designed that support the 802.11a, HiperLAN2, and 802.16a standards [23]–[25], operating in the upper 5 GHz-range band. The test circuit of the shown bipolar LC-VCO is used for verification of the phase-noise model derived using the spectral noise analysis method. It consists of the symmetric LC-tank inductor $L$, two nMOS varactors $C_V$, feedback capacitors $C_A$ and $C_B$, a cross-coupled transistor $Q_1 \cdots Q_2$, and a bias current source $Q_{CS}$, $V_{CC}$ is the supply voltage, $V_{TUNE}$ the varactor tuning voltage, $V_B$ the bias base voltage, and $I_{T,\text{TAIL}}$ the bias current.

**A. Bipolar LC-Oscillator Circuit Parameters**

The test oscillator bias and circuit parameters are optimized for the largest voltage swing (and $k$) around the 5.7 GHz central oscillation frequency, which allows for a comparison of the measured/simulated and calculated (modeled) phase-noise performance.

For a supply voltage of $V_{CC} = 2.2$ V, a transistor base bias voltage $V_B$ between 1.8–1.85 V has been chosen. It allows for both the largest voltage swing of the output signal and the most efficient use of the voltage headroom available, as obtained from the simulations. The maximum voltage swing is estimated from the saturation condition of the transconductor transistors, in order to avoid noise injection of the forward-biased base-collector junctions [15]. This is given by

$$v_{SB,\text{MAX}} \leq 2V_{CC} - V_B + V_{BE} - V_{CE,\text{SAT}} \frac{n}{n+1}$$  \hspace{1cm} (71)$$

where $V_{BE}$ is the forward base-emitter voltage, $V_{CE,\text{SAT}}$ the collector-emitter saturation voltage, $v_{SB,\text{MAX}}$ the maximum voltage swing across the bases of the transconductor. For a capacitor divider ratio or around $n \sim 1.6$, $v_{SB,\text{MAX}}$ of around 0.68 V is expected, corresponding to a small-signal loop gain of around 10.

Compromising between good phase noise, low power consumption, and large frequency tuning range (aiming at the upper...
Fig. 10. Test bipolar LC voltage-controlled oscillator (a) schematic, (b) photomicrograph.

TABLE I
PHASE-NOISE PARAMETERS OF THE BIPOLAR LC-VCO SHOWN IN FIG. 10

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{TK}$</td>
<td>1/340S</td>
</tr>
<tr>
<td>$n$</td>
<td>1.6</td>
</tr>
<tr>
<td>$k$</td>
<td>10</td>
</tr>
<tr>
<td>$c$</td>
<td>0.25</td>
</tr>
<tr>
<td>$L$</td>
<td>1.2nH</td>
</tr>
<tr>
<td>$C_B/C_A$</td>
<td>250fF/100fF</td>
</tr>
<tr>
<td>$f_0$</td>
<td>5.7GHz</td>
</tr>
</tbody>
</table>

802.11a/HIPERLAN/802.16a band), the oscillator circuit parameters have been determined. The 1.2 nH symmetric and differentially shielded LC-tank inductor [$L$ in Fig. 10(a)] has been designed using 4 um thick aluminum top metal. It has two turns, an outer dimension of 190 um, metal width of 10 um, and metal spacing of 5 um. Two n-type MOS varactors with 40 gates complete the LC-tank. Metal-insulator-metal capacitors $C_A$ and $C_B$ are 100 and 250 fF, respectively. Two common-collector output buffers interface the test oscillator and a 50 Ω measurement set-up, each consuming 1.1 mA of current. All the phase-noise relevant circuit parameters are overviewed in Table I.

B. Experimental Results for the Bipolar LC-Oscillator

The chip photomicrograph of the core of the bipolar voltage-controlled oscillator designed is shown in Fig. 10(b). The oscillator core occupies an area of $215 \times 340 \mu m^2$, including buffers. After wirebonding into 32-lead quad packages, the oscillator design was connected to a printed-circuit board with bias and supply line filtering for testing [15].

A frequency tuning range of 600 MHz (5.45 GHz–6.05 GHz) was measured for a 0.9 V tuning voltage range (i.e., between 1.3 and 2.2 V) [9]. The error in the prediction of the oscillation frequency is below 1%. This frequency tuning range covers the upper band of 802.11a/HIPERLAN/802.16a standards, and with additional MOS capacitors in parallel with the LC-tank the operating frequency could be trimmed to cover the complete 5 GHz band.

At around $-12$ dBm output power from a single buffer, the test oscillator achieves a phase noise of around $-106$ dBc/Hz at 1 MHz offset from the 5.7 GHz-band carrier for a current consumption of 4.8 mA and a 2.2 V supply voltage [9]. The spectrum and phase-noise plots of the oscillation signal are shown in Figs. 11 and 12.

As expected, the low phase-noise performance has resulted from the large contribution of noise from the bias current source that is shown in our phase-noise model to be much larger than all other phase-noise contributions (i.e., $g_m$-cell and LC-tank).

The overwhelming contribution of the BCS noise to the phase noise has already been confirmed by the extracted simulations of
Section V-A. The simulations show that for a small-signal loop gain of around 10, the BCS noise accounts for around 85%, the $g_m$-cell noise for around 10%, and the LC-tank noise for 5% of the phase-related noise power (the noise sources considered account for around 90% of the total phase noise).

Numerous noise-optimization procedures have been proposed that reduce noise generated by the LC-oscillator bias current source as the largest phase-noise contributor. As this topic attracts a lot of interest and a number of references can be found in the literature, we only make a brief mention of a result in [9], and refer to [10], [18], [26], [27] for a detailed treatment of other bias-current source noise reduction methods.

The phase-noise model derived in this paper has been used for the design of another LC-oscillator with the phase-noise contribution of the bias current source reduced. By forming a resonance in the emitter of the bias current source transistor at twice the oscillation frequency between an inductor and a base-emitter capacitor [9], a manifold phase noise improvement of the bipolar LC-oscillator under consideration can be realized: 6 dB better phase-noise has been measured from the bipolar LC-oscillator in [9] using this method, resulting in a phase noise of $-112$ dBc/Hz at 1 MHz offset from the 5.7 GHz-band carrier for a current consumption of 4.8 mA and a 2.2 V supply voltage, thereby satisfying the phase-noise requirements of the wireless communication standards in the 5 GHz upper frequency band.

C. Discussion of Experimental Results for the Bipolar LC-Oscillator

In this section we compare the measurement results to the results predicted by the oscillator phase-noise model derived in this paper and the extracted simulation results, for the bipolar LC-VCO.

For the extracted oscillator LC-tank parameters given in Section V-A, the results obtained from the phase-noise model, (67), and the results of simulations and measurement are compared for the largest small-signal loop gain in this section.

The result obtained from the phase-noise model is $-107.4$ dBc/Hz, the result of extracted simulations $-108.6$ dBc/Hz, and the result of measurements $-106.1$ dBc/Hz, all referring to 1 MHz offset from a 5.7 GHz carrier, and current consumption of 4.8 mA.

For the oscillator bias chosen, the simulations predict the best phase noise for a bias current of 4.8 mA. However, the best phase noise of the oscillator has been measured at around 5.4 mA, expected due to a lower than estimated Q-factor of the LC-tank. Correcting for the difference in LC-tank conductance, voltage swing, and noise factor between the measurements and simulations (i.e., accounting for a lower Q-factor of the LC-tank, lower voltage swing, and lower noise factor), calculations and simulations predict a phase noise of around $-106.4$ dBc/Hz and $-107.6$ dBc/Hz at 1 MHz offset from the 5.7 GHz-band carrier, respectively.

For the difference between the measurement and simulation results we may point to the difference in the estimated LC-tank parameters. The noise contributed by the base-current shot noise and base bias circuitry of the transistor as well as the noise of the diode-connected bias current source transistor, which have not been taken into account in the phase-noise model, are another sources of difference. The noise sources considered account for around 90% of the total phase noise, as obtained from the simulations. As already discussed in Section IV-F, the difference between the calculation and simulation results originates from the limited operational bandwidth of the oscillator devices, not accounted for in the phase-noise model developed in this paper. Minor noise sources not considered in the analysis contribute to the difference as well.

The results of the comparative calculation/simulation study have demonstrated the validity of the phase-noise model derived. The formulations derived are intuitive and give insight into the origin of phase noise in bipolar LC-oscillators. They are amenable for design as they describe the oscillator phase-noise performance using electrical circuit parameters.

The measurement results of this section, the simulation results of Section V-A, and the comparative study with the results found in literature of Sections V-B, verify the oscillator phase-noise models of (67) derived for the bipolar LC-VCO shown in Fig. 2.

VII. CONCLUSION

We have used spectral noise analysis to derive an intuitive and complete phase-noise formulation for bipolar LC voltage-controlled oscillators. A phase-noise model of hard-switching bipolar LC-oscillators has been developed as a function of the oscillator circuit parameters. The results obtained from the rigorous phase-noise analyses are valuable to oscillator designers, describing how phase noise is generated in LC-oscillators.

The contributions of all noise sources to the phase noise of the bipolar LC voltage-controlled oscillators have also been determined. It has been shown that the contribution of the bias current source noise to the phase noise is larger than all other noise contributions put together, a factor of around the small-signal loop gain.

The validity of the phase-noise model derived has been tested by comparing the model predictions with the results obtained from the simulations and measurements of the bipolar LC-oscillator designed. The phase-noise model developed in this paper compares favorably with other models found in literature.

REFERENCES


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